Problem 8

Use the preliminary test to decide whether the following series are divergent or require further testing. *Careful:* Do *not* say that a series is convergent; the preliminary test cannot decide this.

$$\sum_{n=1}^{\infty} \frac{\ln n}{n}$$

Solution

Take the limit of the summand as $n \to \infty$, using l'Hôpital's rule where it's appropriate.

$$\begin{aligned}
\lim_{n\to\infty} \frac{\ln n}{n} &\overset{\substack{\infty \\ \overset{}{=} \\ \infty}}{\underset{\text{H}}{}} \lim_{n\to\infty} \frac{\frac{d}{dn}(\ln n)}{\frac{d}{dn}(n)} \\
&= \lim_{n\to\infty} \frac{\frac{1}{n}}{1} \\
&= \lim_{n\to\infty} \frac{1}{n} \\
&= 0
\end{aligned}$$

Since it's zero, no conclusion can be drawn. Further testing is needed.